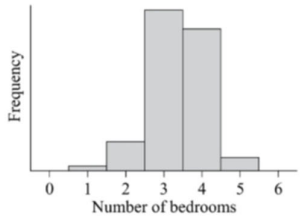
Question 1

QUESTION 10

Simple Familiar
Technology Free
2021

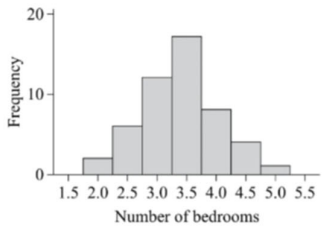
The 2016 Australian census recorded the number of bedrooms per household. The results are summarised in the histogram, as shown. Based on this data, the mean number of bedrooms per household was calculated to be 3.5.



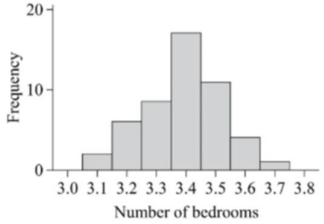
Fifty samples of size 40 were randomly selected from the census data and the sample means recorded.

The histogram that most likely represents the distribution of the sample means is

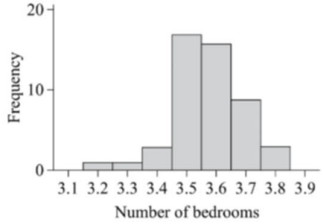
(A)



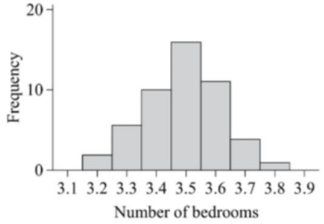
(B)



(C)



(D)



Question 2

QUESTION 7

Simple Familiar
Technology Free
2023

The differential equation for which the solution is a logistic equation of the form $y = \dfrac{a}{b + Ce^{-at}}$ where a, b and C are constants is

(A) $\dfrac{dy}{dt} = 0.25(1 - 0.01t)$

(B) $\dfrac{dy}{dt} = 0.25(1 - 0.01y)$

(C) $\dfrac{dy}{dt} = 0.25t(1 - 0.01t)$

(D) $\dfrac{dy}{dt} = 0.25y(1 - 0.01y)$

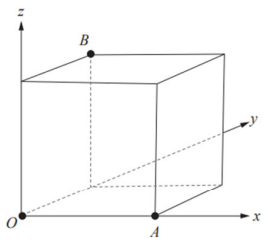
Question 3

QUESTION 14 (6 marks)

Simple Familiar
Technology Free
2023

Consider a cube with three edges positioned along the x-, y- and z-axes on the Cartesian plane as shown. Points O, A and B are vertices of the cube.

Not to scale



a) Given $\overrightarrow{OA} = 2\hat{\boldsymbol{i}}$, determine $\overrightarrow{OB}$. Express your answer in terms of $\hat{\boldsymbol{j}}$ and $\hat{\boldsymbol{k}}$. *[1 mark]*

b) Calculate $\overrightarrow{OA} \times \overrightarrow{OB}$. *[1 mark]*

Consider the triangle formed by joining points O, A and B.

c) Use the result from Question 14b) to determine the area of the triangle. *[2 marks]*

Let points M and N be the midpoints of the triangle's sides OA and OB respectively.

d) Determine $\overrightarrow{MN}$. *[1 mark]*

e) Use the result from Question 14d) to show that the length of AB is twice the length of MN. *[1 mark]*

Question 4

QUESTION 19 (6 marks)

Complex Unfamiliar
Technology Free
2023

Object A is released from the origin with constant velocity, $\boldsymbol{v}_A$, such that its position after t seconds is given by

$$\boldsymbol{r}_A = 2\sqrt{3}t\hat{\boldsymbol{i}} + 3t\hat{\boldsymbol{j}} + 2t\hat{\boldsymbol{k}},\ t \geq 0.$$

At a later time, object B is released from point $P\left(3\sqrt{3}, 6, 0\right)$ and travels towards point $Q\left(5\sqrt{3}, 8, 4\right)$ with constant velocity, $\boldsymbol{v}_B$, such that $|\boldsymbol{v}_B| = \sqrt{2}|\boldsymbol{v}_A|$.

Given that objects A and B collide, determine the time between the release of the two objects.
Assume all positions are given in metres and all velocities are given in metres per second.

Question 5

QUESTION 18 (5 marks)

Complex Unfamiliar
Technology Free
2022

It is proposed that the following expression is divisible by $(1 + \text{cis}(\theta))$ for $n \in Z^+$, $(1 + \text{cis}(\theta)) \neq 0$.

$$\sum_{r=0}^{2n+1} \text{cis}(r\theta)$$

Evaluate the reasonableness of the proposition.