

## Speranza Guesses the QLD Methods External Exam 2022

During the September holidays of 2022, Mr Speranza made 10 Mathematics Videos. In these videos, he did his best to guess what kind of questions might be on the External Exam. Some questions I wrote myself, some were borrowed from Victoria and Western Australia, others were donated by wonderful QLD teachers, and some were written by talented students in my own class.


If you would like to watch the playlist as it was originally made, go here: <https://youtube.com/playlist?list=PL3NIIU3-qaWJI5LVJ3thMEEq7EWFwN4pC>



If you would instead like to attempt it as practise exam, all the questions are in correct exam order below. Solutions are all provided as both video links and QR codes, for ease of access.

Good luck!

### PAPER 1: TECH FREE

	Question	Video Solution
1	<p>If <math>y = a^{b-4x} + 2</math>, where <math>a &gt; 0</math>, then <math>x</math> is equal to</p> <p>A. <math>\frac{1}{4}(b - \log_a(y - 2))</math></p> <p>B. <math>\frac{1}{4}(b - \log_a(y + 2))</math></p> <p>C. <math>b - \log_a\left(\frac{1}{4}(y + 2)\right)</math></p> <p>D. <math>\frac{b}{4} - \log_a(y - 2)</math></p>	 <a href="https://youtu.be/MEY6H-L3M_Q?t=13">https://youtu.be/MEY6H-L3M_Q?t=13</a>

2

If  $\int_1^{12} g(x) dx = 5$  and  $\int_{12}^5 g(x) dx = -6$ , then  $\int_1^5 g(x) dx$  is equal to

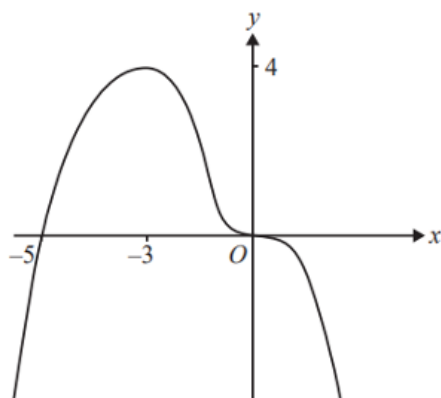
- A. -11  
B. -1  
C. 1  
D. 11



[https://youtu.be/MEY6H-L3M\\_Q?t=191](https://youtu.be/MEY6H-L3M_Q?t=191)

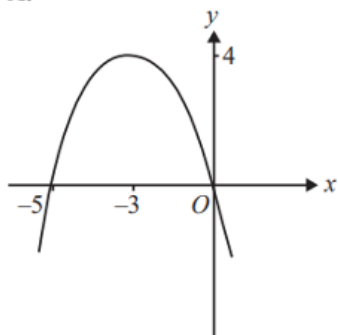
3

The graph of the function  $y = f(x)$  is shown below.

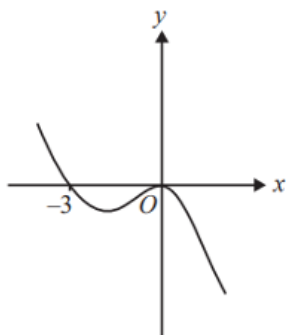


Which of the following could be the graph of the derivative function  $y = f'(x)$ ?

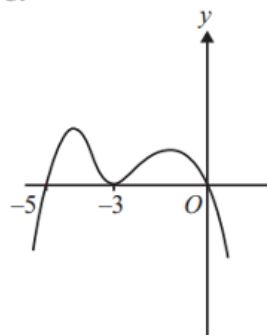
A.



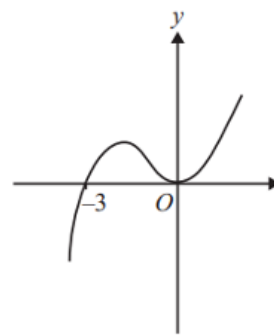
B.






C.



D.



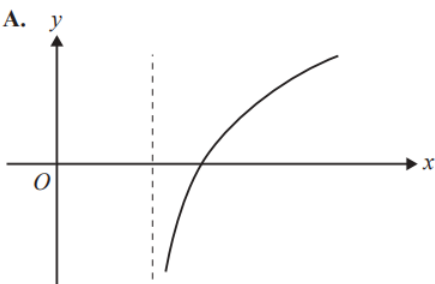
[https://youtu.be/u\\_lEx81er2U?t=26](https://youtu.be/u_lEx81er2U?t=26)

4	<p>The binomial random variable, <math>X</math>, has <math>E(X) = 2</math> and <math>\text{Var}(X) = \frac{4}{3}</math>. <math>\Pr(X = 1)</math> is equal to</p> <p>A. <math>\left(\frac{1}{3}\right)^6</math></p> <p>B. <math>\left(\frac{2}{3}\right)^6</math></p> <p>C. <math>\frac{1}{3} \times \left(\frac{2}{3}\right)^2</math></p> <p>D. <math>6 \times \frac{1}{3} \times \left(\frac{2}{3}\right)^5</math></p> <p>E. <math>6 \times \frac{2}{3} \times \left(\frac{1}{3}\right)^5</math></p>	 <p><a href="https://www.youtube.com/watch?v=u_IEx81er2U&amp;t=232s">https://www.youtube.com/watch?v=u_IEx81er2U&amp;t=232s</a></p>
5	<p>Let <math>k = \int_{-2}^{-1} \frac{1}{x} dx</math>, then <math>e^k</math> is equal to</p> <p>A. <math>\log_e(2)</math></p> <p>B. 1</p> <p>C. 2</p> <p>D. <math>e</math></p> <p>E. <math>\frac{1}{2}</math></p>	 <p><a href="https://www.youtube.com/watch?v=8trO2XGins8&amp;t=18s">https://www.youtube.com/watch?v=8trO2XGins8&amp;t=18s</a></p>
6	<p>The position of an object is given by <math>x = 3t^3 + 5t^2</math> where <math>x</math> represents the position in metres and <math>t</math> is the time in seconds. It's acceleration at 2 seconds is equal to:</p> <p>a. <math>48 \text{ ms}^{-2}</math></p> <p>b. <math>46 \text{ ms}^{-2}</math></p> <p>c. <math>44 \text{ ms}^{-2}</math></p> <p>d. <math>42 \text{ ms}^{-2}</math></p>	 <p><a href="https://www.youtube.com/watch?v=8trO2XGins8&amp;t=139s">https://www.youtube.com/watch?v=8trO2XGins8&amp;t=139s</a></p>

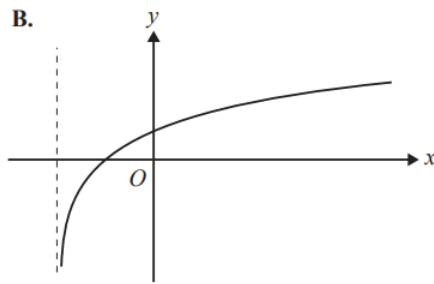
7

Which one of the following could be the graph of  $y = a \log_e (x - b)$  where  $a < 0$  and  $b > 0$ ?

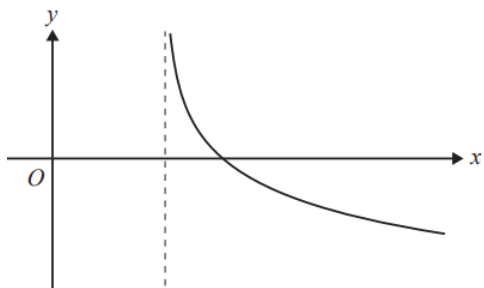
A.



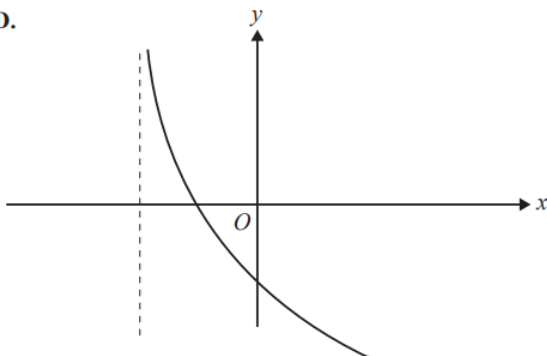
B.



C.



D.



<https://youtu.be/611gZtyMsj4?t=14>

8

The random variable  $X$  has a normal distribution with mean 12 and standard deviation 0.5.

If  $Z$  has the standard normal distribution, then the probability that  $X$  is less than 11.5 is equal to

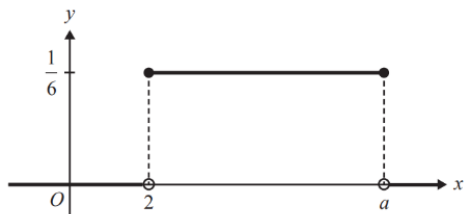
- A.  $\Pr(Z > -1)$
- B.  $\Pr(Z < -0.5)$
- C.  $\Pr(Z > 1)$
- D.  $\Pr(Z \geq 0.5)$
- E.  $\Pr(Z < 1)$



<https://youtu.be/611gZtyMsj4?t=105>

9

The graph of the probability density function of a continuous random variable,  $X$ , is shown below.



If  $a > 2$ , then  $E(X)$  is equal to

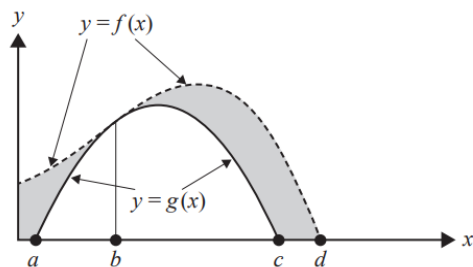
- A. 8
- B. 5
- C. 4
- D. 3
- E. 2



<https://youtu.be/ZeJC7goD3tk?t=18>

10

Consider the graphs of the functions  $f$  and  $g$  shown below.







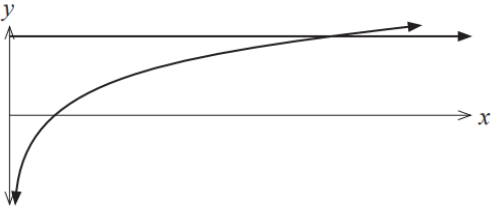



The area of the shaded region could be represented by

- A.  $\int_a^d (f(x) - g(x)) dx$
- B.  $\int_0^d (f(x) - g(x)) dx$
- C.  $\int_0^b (f(x) - g(x)) dx + \int_b^c (f(x) - g(x)) dx$
- D.  $\int_0^a f(x) dx + \int_a^c (f(x) - g(x)) dx + \int_c^d f(x) dx$
- E.  $\int_0^d f(x) dx - \int_a^c g(x) dx$



<https://youtu.be/ZeJC7goD3tk?t=158>

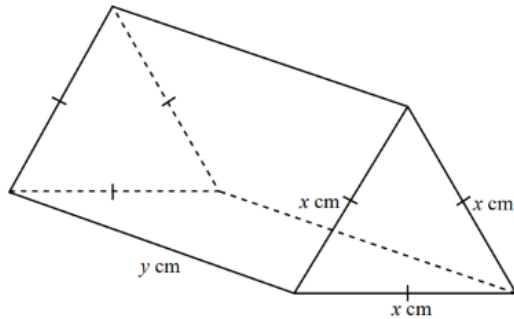
11	<p>Determine the derivatives of (no simplification required):</p> <p>(a) <math>f(x) = 3x^2 \sin(x)</math> (2 marks)</p> <p>(b) <math>y = e^{4 \ln(2x)}</math> (2 marks)</p> <p>(c) <math>g(x) = \frac{(4-x)}{(2x+1)^2}</math> (2 marks)</p>	 <p><a href="https://www.youtube.com/watch?v=MEY6H-L3M_Q&amp;t=486s">https://www.youtube.com/watch?v=MEY6H-L3M_Q&amp;t=486s</a></p>
12	<p>Given that <math>\ln(2) = x</math> and <math>\ln(7) = y</math>:</p> <p>a. Express <math>\ln(14)</math> in terms of <math>x</math> and <math>y</math> (2 marks)</p> <p>b. Express <math>y - 2x + 1</math> in the form <math>\log_a b</math> (2 marks)</p> <p>c. Simplify <math>e^{y-x}</math> (2 marks)</p>	 <p><a href="https://www.youtube.com/watch?v=MEY6H-L3M_Q&amp;t=884s">https://www.youtube.com/watch?v=MEY6H-L3M_Q&amp;t=884s</a></p>
13	<p>A paddock contains 10 tagged cows and 20 untagged cows. Four times each day, one calf is selected at random from the paddock, placed in an observation area and studied, then returned to the paddock.</p> <p>a. Calculate the probability that the number of tagged cows selected on a given day is zero. (2 marks)</p> <p>b. Calculate the probability that at least one tagged cow is selected on a given day. (2 marks)</p> <p>c. Determine the probability that no tagged cows are selected on each of six consecutive days.</p> <p>d. Express your answer in the form <math>\left(\frac{a}{b}\right)^c</math>, where <math>a</math>, <math>b</math> and <math>c</math> are positive integers. (2 marks)</p>	 <p><a href="https://youtu.be/u_IEx81er2U?t=458">https://youtu.be/u_IEx81er2U?t=458</a></p>
14	<p>The graph of the function with the rule <math>f(x) = e^{2x} - 3ke^x + 5</math> intersects the axes at <math>(0,0)</math> and <math>(a,0)</math> and has a horizontal asymptote at <math>y=b</math>. Solve for <math>a</math>, <math>b</math> and <math>k</math>. Give your answers as exact values. (5 marks)</p>	 <p><a href="https://www.youtube.com/watch?v=u_IEx81er2U&amp;t=930s">https://www.youtube.com/watch?v=u_IEx81er2U&amp;t=930s</a></p>

15	<p>(a) Determine a simplified expression for <math>\frac{d}{dx}(x \ln(x))</math>. (2 marks)</p> <p>(b) Use your answer from part (a) to show that <math>\int \ln(x) dx = x \ln(x) - x + c</math>, where <math>c</math> is a constant. (4 marks)</p> <p>The graphs of the functions <math>f(x) = 5</math> and <math>g(x) = \ln(x)</math> are shown below.</p>  <p>(c) Determine the exact area enclosed between the <math>x</math> axis, the <math>y</math> axis and the functions <math>f(x)</math> and <math>g(x)</math>. (4 marks)</p>	 <p><a href="https://youtu.be/8trO2XGins8?t=231">https://youtu.be/8trO2XGins8?t=231</a></p>
16	<p>Determine the equation of the tangent to the graph of <math>f(x) = \sqrt{2} \cos(3x)</math> at point <math>\left(\frac{\pi}{4}, f\left(\frac{\pi}{4}\right)\right)</math>. (4 marks)</p>	 <p><a href="https://youtu.be/8trO2XGins8?t=814">https://youtu.be/8trO2XGins8?t=814</a></p>
17	<p>Two independent samples of different sizes were taken from a population. The first sample had sample size <math>n_1</math> and the second sample had sample size <math>n_2</math>. The sample proportions of males in the samples were the same. When 99% confidence intervals were calculated for each sample, it was found that the corresponding margin of error in the second sample was half that of the first sample.</p> <p>What is the ratio of the two sample sizes, <math>\frac{n_2}{n_1}</math> ? (3 marks)</p>	 <p><a href="https://youtu.be/611gZtyMsi4?t=289">https://youtu.be/611gZtyMsi4?t=289</a></p>

18

A plastic brick is made in the shape of a right triangular prism. The triangular end is an equilateral triangle with side length  $x$  cm and the length of the brick is  $y$  cm.

(6 marks)



The volume of the brick is  $1000 \text{ cm}^3$ .

- Determine an expression for  $y$  in terms of  $x$ .
- Show that the total surface area,  $A \text{ cm}^2$ , of the brick is given by  $A = \frac{4000\sqrt{3}}{x} + \frac{x^2\sqrt{3}}{2}$ .
- Determine the value of  $x$  for which the brick has a minimum total surface area.



<https://youtu.be/611gZtyMsj4?t=599>

19

A farmer grows watermelons. The distribution of the weight of the watermelons is given by the following probability density function, (where  $x$  is the weight of watermelons in kg):

$$\Pr(X = x) = \begin{cases} \frac{\pi}{20} \sin\left(\frac{\pi(x-7)}{10}\right), & 7 < x < 17 \\ 0, & \text{otherwise} \end{cases}$$

All watermelons that weigh more than  $13\frac{2}{3} \text{ kg}$  are moved to a separate batch. From this batch, they are randomly packed into boxes of four.



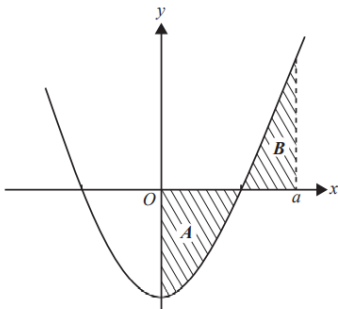

Determine the probability that at least one watermelon in the box will weigh more than  $15\frac{1}{3} \text{ kg}$ . Give your answer in the form  $a - b\sqrt{c}$ . (7 marks)











<https://youtu.be/ZeJC7goD3tk?t=499>



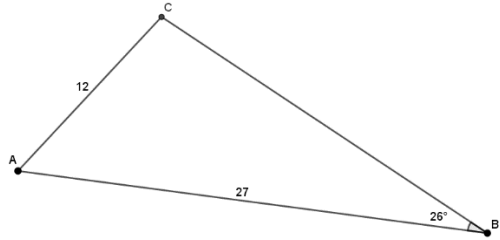








## PAPER 2: TECH ACTIVE

	Question	Video Solution
1	<p>For the continuous random variable <math>X</math> with probability density function</p> $f(x) = \begin{cases} \log_e(x) & 1 \leq x \leq e \\ 0 & \text{elsewhere} \end{cases}$ <p>the expected value of <math>X</math>, <math>E(X)</math>, is closest to</p> <p>A. 0.358 B. 0.5 C. 1 D. 2.097</p>	 <a href="https://youtu.be/bu_CFIEXBxg?t=12">https://youtu.be/bu_CFIEXBxg?t=12</a>
2	<p>The weights of bags of flour are normally distributed with mean 252 g and standard deviation 12 g. The manufacturer says that 40% of bags weigh more than <math>x</math> g.</p> <p>The maximum possible value of <math>x</math> is closest to</p> <p>A. 249.0 B. 251.5 C. 253.5 D. 255.0</p>	 <a href="https://youtu.be/bu_CFIEXBxg?t=112">https://youtu.be/bu_CFIEXBxg?t=112</a>
3	<p>A part of the function, <math>f(x) = x^2 - 4</math>, is shown below.</p>  <p>The area of the region marked <math>A</math> is the same as the area of the region marked <math>B</math>.</p> <p>The exact value of <math>a</math> is</p> <p>A. 0 B. 6 C. <math>\sqrt{6}</math> D. 12 E. <math>2\sqrt{3}</math></p>	 <a href="https://www.youtube.com/watch?v=VvJ3Ls2RJHg&amp;t=14s">https://www.youtube.com/watch?v=VvJ3Ls2RJHg&amp;t=14s</a>

4	<p>Given <math>f(x) = x^4 + 3x^3 - 2x^2 + 5x + 2</math>, <math>f(x)</math> is concave down when:</p> <ol style="list-style-type: none"> <li><math>-1.7 &lt; x &lt; 0.2</math></li> <li><math>x &lt; -2.77</math></li> <li><math>x \leq -2.77</math></li> <li><math>-1.7 \leq x \leq 0.2</math></li> </ol>	 <a href="https://www.youtube.com/watch?v=VvJ3Ls2RJHg&amp;t=313s">https://www.youtube.com/watch?v=VvJ3Ls2RJHg&amp;t=313s</a>
5	<p>A fair coin is tossed 10 times.</p> <p>The probability, correct to four decimal places, of getting 8 or more heads is</p> <ol style="list-style-type: none"> <li>0.0039</li> <li>0.0107</li> <li>0.0547</li> <li>0.9453</li> <li>0.9893</li> </ol>	 <a href="https://youtu.be/0ff236UL3cg?t=14">https://youtu.be/0ff236UL3cg?t=14</a>
6	<p>The times (in minutes) taken for students to complete a university test are normally distributed with a mean of 200 minutes and standard deviation 10 minutes.</p> <p>The proportion of students who complete the test in less than 208 minutes is closest to</p> <ol style="list-style-type: none"> <li>0.200</li> <li>0.212</li> <li>0.758</li> <li>0.788</li> <li>0.800</li> </ol>	 <a href="https://youtu.be/0ff236UL3cg?t=81">https://youtu.be/0ff236UL3cg?t=81</a>
7	<p>Let <math>R</math> be the region enclosed by the graph of <math>y = -2xe^{-2x}</math>, the <math>x</math>-axis, and the lines <math>x = -1</math> and <math>x = 1</math>. The area of <math>R</math> is closest to</p> <ol style="list-style-type: none"> <li>3.9</li> <li>4.487</li> <li>3.893</li> <li>4.19</li> </ol>	 <a href="https://youtu.be/Ec04CI7ACZE?t=16">https://youtu.be/Ec04CI7ACZE?t=16</a>

8	<p>The limit of <math>\frac{14^h - 1}{h}</math> as <math>h</math> approaches 0 is closest to:</p> <p>a) <math>7.145 \times 10^9</math>  b) <math>-1 \times 10^{11}</math>  c) 2.65  d) 0.264</p>	 <a href="https://youtu.be/Ec04CI7ACZE?t=195">https://youtu.be/Ec04CI7ACZE?t=195</a>
9	<p>An isosceles triangle has an area of <math>60\text{cm}^2</math> and one of its internal angles is <math>98^\circ</math>.  The length of one of the equal sides is closest to:</p> <p>A. 11.01  B. 7.78  C. 5.50  D. 13.52</p>	 <a href="https://youtu.be/tU4t0egihuk?t=18">https://youtu.be/tU4t0egihuk?t=18</a>
10	<p>In a random sample of 600 Mathematical Methods students, 70 said that they watched Speranza's "Guess the Methods Exams" videos. The 85% confidence interval for the proportion of all students who watched his videos is closest to:</p> <p>A. (0.0910, 0.1424)  B. (0.0978, 0.1355)  C. (0.0951, 0.1382)  D. (0.0829, 0.1504)</p>	 <a href="https://youtu.be/tU4t0egihuk?t=196">https://youtu.be/tU4t0egihuk?t=196</a>
11	<p>The following function is a probability density function on the given interval:</p> $f(x) = \begin{cases} ax^2(x-2) & \text{for } 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$ <p>(a) Find the value of <math>a</math>. (3 marks)</p> <p>(b) Find the probability that <math>x \geq 1.2</math>. (2 marks)</p> <p>(c) Find the median of the distribution. (2 marks)</p>	 <a href="https://youtu.be/bu_CFIEXBxg?t=245">https://youtu.be/bu_CFIEXBxg?t=245</a>

12	<p>The weight, <math>X</math>, of chicken eggs from a farm is normally distributed with mean 60 g and standard deviation 5 g. Eggs with a weight of more than 67 g are classed as 'jumbo'.</p> <p>(a) What proportion of eggs from the farm are 'jumbo'? (2 marks)</p> <p>(b) What proportion of 'jumbo' eggs are less than 75 g in weight? (3 marks)</p> <p>(c) The heaviest 0.05% of eggs fetch a higher price. What is the minimum weight of these eggs? (2 marks)</p>	 <a href="https://youtu.be/bu_CFIEXBxg?t=509">https://youtu.be/bu_CFIEXBxg?t=509</a>
13	<p>Amy is an expert pistol shooter. The probability that Amy hits the centre of the target on each attempt is 0.12. Determine the least number of attempts Amy must make in order to ensure the probability of hitting at least once is 85%.</p>	 <a href="https://youtu.be/VvJ3Ls2RJHg?t=481">https://youtu.be/VvJ3Ls2RJHg?t=481</a>
14	<p>Determine the length of the longest side in the triangle shown.</p> 	 <a href="https://www.youtube.com/watch?v=VvJ3Ls2RJHg&amp;t=875s">https://www.youtube.com/watch?v=VvJ3Ls2RJHg&amp;t=875s</a>
15	<p>Fermium-257 is a radioactive substance whose decay rate can be modelled by the formula <math>P = P_0 e^{kt}</math>, where <math>P</math> is the mass in grams and <math>t</math> is measured in days and <math>P_0</math> = original amount and <math>k</math> is a constant. The time taken to decay to half of the original amount is known as half-life. The half-life of Fermium-257 is 100.5 days.</p> <p>(a) Determine the value of <math>k</math> to three significant figures. (3 marks)</p> <p>(b) How many days will it take for 100 grams of the substance to first decay below five grams? (2 marks)</p> <p>(c) Determine the rate of change of the amount of Fermium on the day found in part (b). (2 marks)</p>	 <a href="https://youtu.be/0ff236UL3cg?t=240">https://youtu.be/0ff236UL3cg?t=240</a>

16	<p>The waiting times at a Perth Airport departure lounge have been found to be normally distributed. It is observed that passengers wait for less than 55 minutes, 5% of the time, while there is a 13% chance that the waiting times will be greater than 100 minutes.</p> <p>(a) Determine the mean and standard deviation for the waiting times at Perth Airport departure lounge. (5 marks)</p> <p>(b) Determine the probability that the waiting time will be between 75 and 90 minutes. (1 mark)</p>	 <a href="https://youtu.be/Off236UL3cg?t=946">https://youtu.be/Off236UL3cg?t=946</a>
17	<p>The displacement in centimetres of a particle from the point O in a straight line is given by <math>x(t) = \frac{1}{3} \left( \frac{t}{2} - 4 \right)^2 - 2</math> for <math>0 \leq t \leq 10</math>, where <math>t</math> is measured in seconds.</p> <p>Calculate the:</p> <p>(a) time(s) that the particle is at rest. (2 marks)</p> <p>(b) displacement of the particle during the fifth second. (2 marks)</p> <p>(c) maximum speed of the particle and the time when this occurs. (2 marks)</p> <p>(d) total distance travelled in the first 10 seconds. (2 marks)</p>	 <a href="https://youtu.be/Ec04CI7ACZE?t=418">https://youtu.be/Ec04CI7ACZE?t=418</a>
18	<p>A recent study tested how familiar Australian primary school students are with Australia's location on a world map. For the study, researchers randomly selected 50 primary school students, and each student was given 30 seconds to locate Australia on a world map. A student was considered successful if they managed to correctly identify Australia on the world map within the time constraint. (6 marks)</p> <p>Calculating a 95% confidence interval, the researchers determined the upper bound of the confidence interval as being 0.844.</p> <p>Determine the lower bound of the confidence interval.</p>	 <a href="https://youtu.be/Ec04CI7ACZE?t=1452">https://youtu.be/Ec04CI7ACZE?t=1452</a>
19	<p>Consider the following information when completing this question:</p> <p>The length of a curve <math>y = f(x)</math> over the interval <math>[a, b] = \int_a^b \sqrt{1 + (f'(x))^2} dx</math></p> <p>A bumpy road leading from the peak to the base of a mountain can be modelled by the function <math>f(x) = \frac{3}{2} \cos\left(\frac{\pi x}{60}\right) - \frac{x}{50} + 200</math> where <math>x</math> is horizontal distance in metres from the peak and <math>f(x)</math> is vertical distance from the base in metres. The council is proposing to smooth out the road, so that it is a straight line instead of bumpy.</p> <p>Determine how much shorter the road will be once work is complete. (6 marks)</p>	 <a href="https://youtu.be/tU4t0egihuk?t=299">https://youtu.be/tU4t0egihuk?t=299</a>

20

People are given a maximum of six hours to complete a puzzle. The time spent on the puzzle, in hours, can be modelled using the continuous random variable  $X$  which has probability density function ( 6 marks)

$$f(x) = \begin{cases} \frac{2x}{(x^2+4)\ln(10)} & , 0 \leq x \leq 6 \\ 0, & elsewhere \end{cases}$$

The Intelligence Quotient (IQ) scores of people are normally distributed with a mean of 100 and standard deviation of 15.

It has been observed that the puzzle is generally completed more quickly by people with a high IQ.

It is known that 80% of people with an IQ greater than 130 can complete the puzzle in less than two hours.

A person chosen at random can complete the puzzle in less than two hours.

What is the probability that this person has an IQ greater than 130? Give your answer correct to three decimal places.



<https://youtu.be/tU4t0egihuk?t=881>