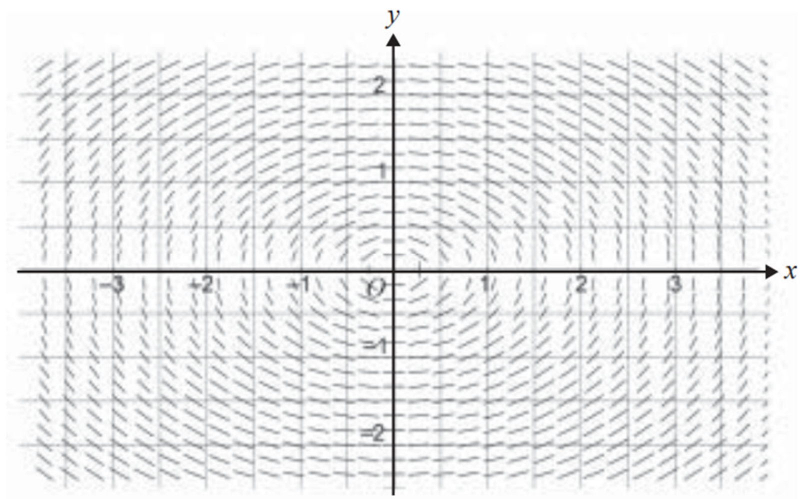
In the lead up to the 2022 External Specialist Exams, Mr Speranza searched every past paper from every State in Australia to create the ultimate revision exam for QLD. This is that paper. If you would like to watch the videos in the order in which they were released, you can visit the playlist here: [Speranza Revises for the Specialist Exam - YouTube]

Otherwise, attempt the questions below and see how you go. Click the individual youtube links next to each question to see how you went.

Paper 1: Tech Free

1	The direction (slope) field for a certain first order differential equation is shown above.  The differential equation could be A. $\frac{dy}{dx} = \frac{x^2}{2} + y^2$ B. $\frac{dy}{dx} = x^2 + \frac{y^2}{2}$ C. $\frac{dy}{dx} = -\frac{x}{2y}$ D. $\frac{dy}{dx} = -\frac{y}{2x}$ E. $\frac{dy}{dx} = \frac{x}{2y}$	https://youtu.be/FHTl0FLcswk?t=18
2	Which of the following is a vector equation of the line joining the points $A(4,2,5)$ and $B(-2,2,1)$? A. $\boldsymbol{r} = \begin{pmatrix}4\\2\\5\end{pmatrix} + \lambda\begin{pmatrix}1\\2\\3\end{pmatrix}$ B. $\boldsymbol{r} = \begin{pmatrix}4\\2\\5\end{pmatrix} + \lambda\begin{pmatrix}3\\0\\2\end{pmatrix}$ C. $\boldsymbol{r} = \begin{pmatrix}1\\2\\3\end{pmatrix} + \lambda\begin{pmatrix}4\\2\\5\end{pmatrix}$ D. $\boldsymbol{r} = \begin{pmatrix}3\\0\\2\end{pmatrix} + \lambda\begin{pmatrix}4\\2\\5\end{pmatrix}$	https://youtu.be/FHTl0FLcswk?t=508

3	The volume of water $V\ m^3$ in a cylindrical tank when it is filled to a depth of h metres is given by $V = 4h$. Water flows into the tank at a rate of $0.2m^3$ per minute and leaks out at a rate of $0.01\sqrt{h}\ m^3$ per minute. The differential equation, which when solved would enable h to be expressed in terms of t, is A. $\frac{dh}{dt} = 0.2 - 0.01\sqrt{h}$ B. $\frac{dh}{dt} = 4(0.2 - 0.01\sqrt{h})$ C. $\frac{dh}{dt} = \frac{20-\sqrt{h}}{400}$ D. $\frac{dh}{dt} = \frac{400}{20-\sqrt{h}}$ E. $\frac{dh}{dt} = 20 - \frac{400}{\sqrt{h}}$	https://youtu.be/uhrKhigNVVI
4	The distance between the two points z and $-\bar{z}$ in the complex plane is given by: A. $2\ Re(z)$ B. $2\ Im\ (z)$ C. $2\|z\|$ D. $2\ Re(z) + 2\ Im(z)$ E. $2Arg(z)$	https://youtu.be/uhrKhigNVVI?t=231
5	Which expression is equal to $\int x^5 e^{7x}\, dx$? A. $\frac{1}{7}x^5e^{7x} - \frac{5}{7}\int x^4e^{7x}\, dx$ B. $\frac{1}{7}x^5e^{7x} - \frac{5}{7}\int x^5e^{7x}\, dx$ C. $\frac{5}{7}x^4e^{7x} - \frac{5}{7}\int x^4e^{7x}\, dx$ D. $\frac{5}{7}x^4e^{7x} - \frac{5}{7}\int x^5e^{7x}\, dx$	https://youtu.be/GRxo70k8h4A?t=17

6	A 5 kg mass has an initial velocity of $4ms^{-1}$. The mass increases its speed by accelerating in a straight line at a constant rate of $2ms^{-2}$. After travelling 21 metres the magnitude of the momentum of the mass in $kg\ ms^{-1}$ is A. 10 B. 20 C. 40 D. 50 E. 60	https://youtu.be/GRxo70k8h4A
7	Which of the following integrals is equivalent to $\int \sin^2 3x\ dx$? A. $\int \frac{1+cos6x}{2}\ dx$ B. $\int \frac{1-cos6x}{2}\ dx$ C. $\int \frac{1+sin6x}{2}\ dx$ D. $\int \frac{1-sin6x}{2}\ dx$	https://youtu.be/m-wGEbV47Pc?t=20
8	When using proof by mathematical induction to show that $(1+i)^{4n} = (-4)^n$ where $n \in \mathbb{Z}^+$, the inductive step requires the proof of A. $(1+i)^4 = (-4)^1$ B. $(1+i)^{4k} = (-4)^k$ C. $(1+i)^{4k+1} = (-4)^{k+1}$ D. $(1+i)^{4k+4} = (-4)^{k+1}$	https://youtu.be/m-wGEbV47Pc?t=164
9	The polynomial $P(z)$ has real coefficients. Four of the roots of the equation $P(z) = 0$ are $z = 0,\ z = 1,\ z = 1-2i,\ z = 1+2i$ and $z = 3i$ The minimum number of roots that the equation $P(z) = 0$ could have is A. 4 B. 5 C. 6 D. 7 E. 8	https://youtu.be/U2ob7_jWqKU?t=15

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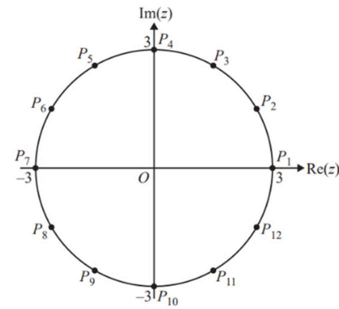
On the argand diagram below, the twelve points, $P_1, P_2, P_3, \dots, P_{12}$ are evenly spaced around the circle of radius 3.



The points which represent complex numbers such that $z^3 = -27i$ are

A. P_{10} only
B. P_4 only
C. P_2, P_6, P_{10}
D. P_3, P_7, P_{11}
E. P_4, P_8, P_{12}

</td><td>https://youtu.be/U2ob7_jWqKU?t=95</td></tr>
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Find the equation of the tangent to the curve $x^3 - 2x^2y + 2y^2 = 2$ at the point $P(2,3)$

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Points A, B have respective position vectors $\begin{pmatrix} 4 \\ 0 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ -2 \\ 5 \end{pmatrix}$

a) Determine the cartesian equation for the sphere that has $\overline{AB}$ as its diameter.

If point O is the origin, consider the plane that contains the vectors $\overline{OA}$ and $\overline{OB}$

b) Determine the vector equation for this plane in the form $\boldsymbol{r}.\boldsymbol{n} = c$

</td><td>https://youtu.be/FHTl0FLcswk?t=1055</td></tr>
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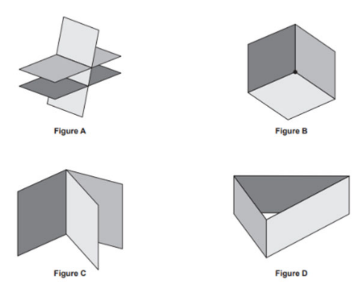
Consider the following system of equations where m is a non-zero real number.

$$x + y = 0$$
$$mx + z = m^2 - 1$$
$$mx + 2my + (3 - m^2)z = 0$$

a) Write this system of equations as an augmented matrix.
b) Using clearly stated row operations, show that the system in part (a) reduces to:

$$\begin{bmatrix} 1 & 1 & 0 & : & 0 \\ 0 & m & -1 & : & (1 - m^2) \\ 0 & 0 & (m^2 - 4) & : & (1 - m^2) \end{bmatrix}$$



c) i) State a value of m for which there is a unique solution.
ii) Which figure below best represents the solution to this system for $m = -2$.

</td><td>https://youtu.be/uhrKhigNVVI?t=428</td></tr>
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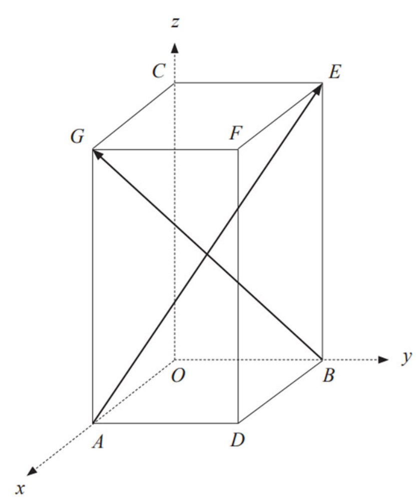
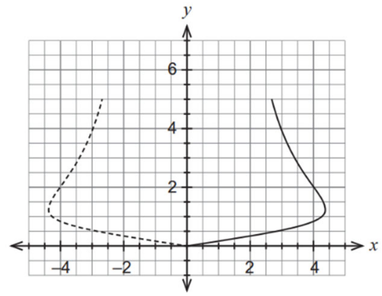
14	Sketch and clearly identify the region of the Argand plane that represents the set of complex numbers $\left\{z: \frac{\pi}{4} \leq Arg(z) \leq \frac{\pi}{3}\right\} \cap \{z: \|z-1\| \leq 1\}$. You should determine and show any key points.	https://youtu.be/uhrKhigNVVI?t=1082
15	Evaluate $\int_{-1}^{7} \frac{3x}{\sqrt{x+2}} dx$ exactly using the substitution $u = \sqrt{x+2}$	https://youtu.be/GRxo70k8h4A?t=406
16	Consider the function $P(z) = z^4 - 2z^3 + 14z^2 - 8z + 40$, defined over the complex numbers. a) Show that $(z - 2i)$ is a factor of $P(z)$ b) Hence or otherwise, solve the equation $P(z) = 0$, giving solutions in the form $a + bi$	https://youtu.be/GRxo70k8h4A?t=718
17	Use mathematical induction to prove that $$4 + 4^2 + 4^3 + \cdots + 4^n = \frac{4}{3}(4^n - 1)$$ Where n is a positive integer.	https://youtu.be/m-wGEbV47Pc?t=328
18	A right rectangular prism, with square base $OADB$, is shown. Point O is the origin and points A, B, C have respective position vectors $\begin{pmatrix}4\\0\\0\end{pmatrix}, \begin{pmatrix}0\\4\\0\end{pmatrix}, \begin{pmatrix}0\\0\\c\end{pmatrix}$ where $c > 0$.  a) Determine, in terms of c, the: i) Vector equation for the line containing points A and E. ii) Cartesian equation for the plane $ADEC$ In general, the main diagonals $\overline{AE}, \overline{BG}$ are not perpendicular to each other. b) Determine the value of c so that the main diagonals of the prism are perpendicular to each other.	https://youtu.be/m-wGEbV47Pc?t=724

19	The top part of a wine glass is modelled by rotating the graph $x^2 = y^2(36 - x^2y)$ from $y = 0$ to $y = 5$ about the y axis as shown below. Dimensions are measured in centimetres. Determine the exact volume $V\ cm^3$.  	https://youtu.be/U2ob7_jWqKU?t=349
20	Consider the complex equation $z^n - 1 = 0$, where n is any positive integer $n \geq 3$. If the roots are designated $z_0, z_1, z_2, \ldots, z_{n-1}$, then determine the exact value for the product of the roots, $p = z_0 \times z_1 \times z_2 \times \ldots \times z_{n-1}$.	https://youtu.be/U2ob7_jWqKU?t=918

Paper 2: Tech Active

1	The position vectors of two moving particles are given by $\boldsymbol{r_1}(t) = (2+4t^2)\boldsymbol{i} + (3t+2)\boldsymbol{j}$ and $\boldsymbol{r_2}(t) = (6t)\boldsymbol{i} + (4+t)\boldsymbol{j}$, where $t \geq 0$. The particles will collide at a) $3\boldsymbol{i} + 3.5\boldsymbol{j}$ b) $6\boldsymbol{i} + 5\boldsymbol{j}$ c) $3\boldsymbol{i} + 4.5\boldsymbol{j}$ d) $0.5\boldsymbol{i} + \boldsymbol{j}$ e) $5\boldsymbol{i} + 6\boldsymbol{j}$	https://youtu.be/X3iKaI6eAQs?t=22
2	A random sample of 100 bananas from a given area has a mean mass of 210 grams and a standard deviation of 16 grams. Assuming the standard deviation obtained from the sample is a sufficiently accurate estimate of the population standard deviation, an approximate 95% confidence interval for the mean mass of bananas produced in this locality is given by: A. (178.7, 241.3) B. (206.9,213,1) C. 209.2,210.8) D. (205.2,214.8) E. (194,226)	https://youtu.be/X3iKaI6eAQs?t=193
3	Let $\boldsymbol{u} = \boldsymbol{i} + \boldsymbol{j}$ and $\boldsymbol{v} = \boldsymbol{i} + 2\boldsymbol{j} + 2\boldsymbol{k}$. The angle between the vectors u and v is A. $0°$ B. $45°$ C. $30°$ D. $22.5°$ E. $90°$	https://youtu.be/ZS6SLYtCm98?t=20

4	If $z = \frac{3+4i}{1+2i}$, the imaginary part of z is A. -2 B. $-\frac{2}{5}i$ C. $-\frac{2}{5}$ D. $-2i$ E. 2	https://youtu.be/ZS6SLYtCm98?t=279
5	Bottles of a particular brand of soft drink are labelled as having a volume of $1.25\ L$. The machines filling the bottles deliver a volume that is normally distributed with a mean of $1.26\ L$ and a standard deviation of $0.01\ L$. The probability that six bottles have a mean volume that is at least the labelled volume of $1.25\ L$ is closest to A. 0.5968 B. 0.8413 C. 0.9750 D. 0.9772 E. 0.9928	https://youtu.be/Ox-OCu3z87I?t=15
6	A 95% confidence interval for the mean height μ, in centimetres, of a random sample of 36 Irish setter dogs is $58.42 < \mu < 67.31$ The standard deviation of the height of the population of Irish setter dogs, in centimetres, correct to two decimal places, is A. 2.26 B. 2.27 C. 13.60 D. 13.61 E. 62.87	https://youtu.be/Ox-OCu3z87I?t=171

<table>
<tr><td>7</td><td>A body moves in a straight line such that its velocity $v\ ms^{-1}$ is given by $v = 2\sqrt{1-x^2}$, where x is its displacement from the origin.
The acceleration of the body in ms^{-2} is given by
A. $\frac{-2x}{\sqrt{1-x^2}}$
B. $-2x$
C. $\frac{2}{\sqrt{1-x^2}}$
D. $2(1-2x)$
E. $-4x$</td><td>https://youtu.be/UDELowQEW4E?t=12</td></tr>
<tr><td>8</td><td>The win/loss results after a soccer competition involving four teams are represented in the matrix M shown below.
$$\mathbf{M} = \begin{array}{cc} & \text{Losing teams} \\ & \begin{array}{cccc} A & B & C & D \end{array} \\ \text{Winning teams} \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 2 & 0 & 1 \\ 2 & 1 & 1 & 0 \end{bmatrix} \end{array}$$
Key: Team A defeated Team B, drew with Team C and lost to Team D.
Given the model $M + M^2 + M^3$ to rank the teams, determine their final positions from first to fourth.
A. C,A,D,B
B. C,D,A,B
C. D,A,C,B
D. D,C,A,B</td><td>https://youtu.be/UDELowQEW4E?t=126</td></tr>
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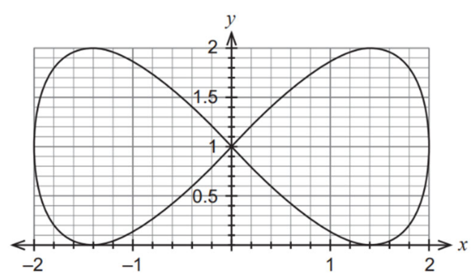
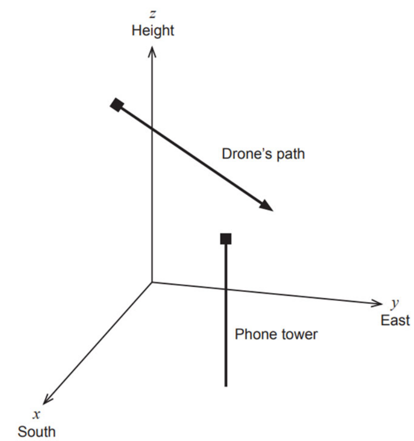
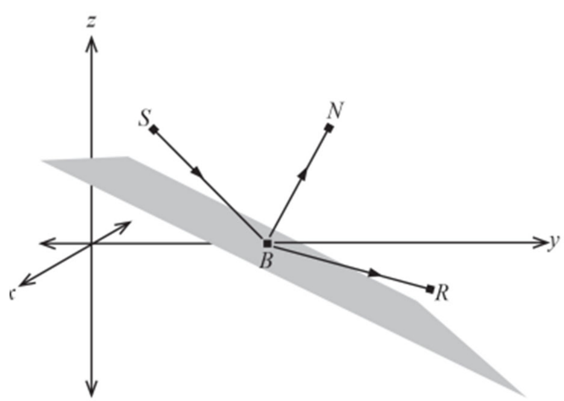
9	The amount of a drug, x mg, remaining in a patient's bloodstream t hours after taking the drug is given by the differential equation $\frac{dx}{dt} = -0.15x$ The number of hours need for the amount x to halve is A. $2\log_e\left(\frac{20}{3}\right)$ B. $\frac{20}{3}\log_e(2)$ C. $2\log_e(15)$ D. $15\log_e\left(\frac{3}{2}\right)$ E. $\frac{3}{2}\log_e(200)$	https://youtu.be/eoF8UguFOOs?t=17
10	The volume of the solid of revolution formed by rotating the graph of $y = \sqrt{9-(x-1)^2}$ about the x-axis is given by A. $4\pi(3)^2$ B. $\pi\int_{-3}^{3}(9-(x-1)^2)\,dx$ C. $\pi\int_{-2}^{4}\left(\sqrt{9-(x-1)^2}\right)dx$ D. $\pi\int_{-2}^{4}(9-(x-1)^2)^2\,dx$ E. $\pi\int_{-4}^{2}(9-(x-1)^2)\,dx$	https://youtu.be/eoF8UguFOOs?t=208
11	The path of a particle is shown below. This particle moves so that its position vector $\boldsymbol{r}(t)$ is given by $\boldsymbol{r}(t) = \begin{pmatrix} -2\cos\left(\frac{t}{2}\right) \\ 1-\sin(t) \end{pmatrix}$ metres, where t is the number of seconds the particle has been in motion.  a) Determine the starting position of the particle and mark this as point A on the diagram above. b) Determine the initial velocity of the particle and illustrate this on the diagram above. c) Determine the cartesian equation for the path of the particle.	https://youtu.be/X3iKaI6eAQs?t=275

12	A small drone is launched and, after hovering in an initial position, it flies in a straight line under the control of its operator. The position of the drone from the operator is given by $r(t) = \begin{pmatrix} 100 + 0.5t \\ 0.6t \\ 50 - 0.02t \end{pmatrix}$ metres, where t is the time in seconds it has been flying in a straight line.  The top of a mobile phone tower is positioned at $200\boldsymbol{i} + 150\boldsymbol{j} + 30\boldsymbol{k}$ relative to the operator i.e. the mobile phone tower is 30 metres tall. a) After two minutes of flight, how high is the drone above the ground? b) Write the expression for the position vector of the drone from the top of the phone tower after t seconds. The operator knows that the drone will not strike the mobile phone tower. However, the operator does not know that the drone will cause interference when it is less than 50 metres from the top of the tower. c) Determine whether the drone will cause interference to the mobile phone tower and, is so, for how long will this occur, correct to the nearest second.	https://youtu.be/X3iKaI6eAQs?t=822
13	The horizontal displacement of a Ferris wheel cabin exhibits simple harmonic motion. The maximum horizontal speed is $\frac{\pi}{2}$ metres per second and its period of motion is exactly 60 seconds. Let $x(t) = A\cos(nt)$ be the horizontal displacement after t seconds. a) Determine the values of A and n. b) Determine the horizontal acceleration, correct to the nearest 0.001 m/s^2, when the horizontal displacement is 10 metres.	https://youtu.be/ZS6SLYtCm98?t=432
14	Solve the equation $z^4 = 8\sqrt{3} + 8i$ giving exact solutions in the form $r cis\theta$ where $-\pi < \theta \leq \pi$.	https://youtu.be/ZS6SLYtCm98?t=905

15	A four metre long water tank, open at the top, is in the shape of a triangular prism. The triangular face is a right isosceles triangle with congruent sides of one metre length. Initially the tank is completely full with water, but it develops a leak and loses water at a constant rate of 0.08 cubic metres per hour. Let $h =$ the depth of water, in metres, in the tank after t hours. a) Show that the volume of water in the tank V cubic metres, is given by the expression $V(h) = 4h^2$. b) Determine the rate of change of the depth, correct to the nearest 0.01 metres per hour, when the depth is 0.6 metres. Assume that the rate of leakage stays constant at 0.08 cubic metres per hour. c) Show that the differential equation that relates $\frac{dh}{dt}$ with the depth h is given by $\frac{dh}{dt} = -\frac{1}{100h}$. d) Hence determine the relationship for the depth h at any time t hours.	https://youtu.be/Ox-OCu3z87I?t=282
16	On a certain nature reserve there are initially 10 pairs of nesting black cockatoos. One theory suggests that the number N, of the nesting pairs after t years will satisfy the differential equation $$\frac{dN}{dt} = \frac{N}{180}(100 - N).$$ a) Show that initially the rate of increase of N is 5 per year. b) Interpret what happens as $N \to 100$. c) Determine $N(t)$. d) After how many years does the population of black cockatoos reach 90% of its maximum.	https://youtu.be/Ox-OCu3z87I?t=896

17	A research student has collected fertility and survival rates for a certain endangered species over a number of years. A small group of this species has been moved into a secure property. The Leslie matrix for the survival rates of the species is $$L = \begin{pmatrix} 0 & 0.1 & 0.8 & 0.5 \\ 0.95 & 0 & 0 & 0 \\ 0 & 0.83 & 0 & 0 \\ 0 & 0 & 0.64 & 0 \end{pmatrix}$$ The female population of the species moved into the property at the start of Year 1 is $$N_1 = \begin{pmatrix} 5 \\ 50 \\ 25 \\ 20 \end{pmatrix}$$ a) Explain the meaning of the number 0.8 in the Leslie matrix. b) Calculate the expected total female population in the property at the start of Year 2. c) Determine the expected total population in the property at the start of Year 11, based on 55% of the population being female.	https://youtu.be/UDELowQEW4E?t=207
18	A laser pointer at point S directs a highly focused beam of light towards a mirror. The beam bounces off the mirror at point B and is then reflected away from the mirror toward point R. The mirror's surface is given by the equation $\boldsymbol{r} \cdot (\boldsymbol{j} + 2\boldsymbol{k}) = 9$ and the laser pointer is positioned at point S with position vector $-2\boldsymbol{i} + 3\boldsymbol{j} + 6\boldsymbol{k}$. The laser pointer is held so that the beam is pointed in the direction $\boldsymbol{d}_1 = \boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k}.$  a) Determine the position vector for point B. The laser beam is reflected away from the mirror so that: ● the angle of the incoming beam $\overrightarrow{SB}$ to the normal of the mirror is equal to the angle of the reflected beam $\overrightarrow{BR}$ to the normal of the mirror i.e. $s\angle SBN = s\angle NBR$ ● the incoming beam $\overrightarrow{SB}$, the normal of the mirror and the reflected beam $\overrightarrow{BR}$ are all contained in one plane. Let $\hat{\boldsymbol{d}}_2$= the unit vector in the direction of the reflected beam $\overrightarrow{BR}$ BR → i.e. $\lvert\hat{\boldsymbol{d}}_2\rvert = 1$ b) Determine the unit vector $\hat{\boldsymbol{d}}_2$ giving components correct to 0.01.	https://youtu.be/eoF8UguFOOs?t=544