EXTENDED MODELLING AND PROBLEM-SOLVING STUDENT STYLE GUIDE

Genre considerations:

Length (up to 10 pages excluding appendices) 2000 word limit absolute maximum (discuss this limit with teacher)

1. Introduction:

In this section, enough information must be given so that the report can be read and understood **WITHOUT** referring to the assignment sheet or question. A teacher or student from another school should be able to start reading here and understand what was on the task sheet.

Include:

- The context of the task.
- Any relevant information from the task sheet.

If you do this introduction well, you should hit this criteria from the ISMG.

The student work has the following characteristics:	Marks
• correct use of appropriate technical vocabulary, procedural vocabulary and conventions to develop the response • coherent and concise organisation of the response, appropriate to the genre, including a suitable introduction, body and conclusion, which can be read independently of the task sheet.	3–4

2. Considerations

The following section is designed to allow students to fulfil the following criteria from the ISMG.

The student work has the following characteristics:	Marks
• documentation of appropriate assumptions • accurate documentation of relevant observations • accurate translation of all aspects of the problem by identifying mathematical concepts and techniques.	3–4

2.1 Observations

- In this section, students should list all observations they have made. This is often done in dot point form.
- An observation is something the student has observed in the reading of the question or in researching the topic.
- If you have looked something up, **use in text referencing to show your source.** Showing sources is one of the best ways to show "documentation".

The QCAA have this to say from the 2022 Mathematical Methods Subject report.

> – responses that demonstrate 'accurate documentation of relevant observations' provide evidence to support observations (data or information) used in a student's model/solution. This can be demonstrated by explaining how the observations were collected, the source of the observations, what made the observations valid and reliable, or identifying a specific feature of an observation that made it relevant to the model/solution, e.g. from the plotted data points it is clearly observed that there is one turning point at (x, y) and that the curve is symmetrical about that turning point

Each year the QCAA publishes samples of EXEMPLARY student work. Here are the 2022 samples of observations from Specialist Mathematics students.

Observations

1. It was observed that the male-to-female ratio in Western Australia is 102 males for every 100 females (McCrindle, 2014). This observation directly impacts the mathematical model as the initial total population must divide by this ratio only to consider the female Western Australian population. This is relevant as males do not reproduce any offspring and cannot be factored into the Leslie matrix.

2. It was observed that Western Australia takes 30% of immigrants into Australia each year (Australian Bureau of Statistic, 2021). Further, 12,706 to 18,200 immigrants settled in Australia during 2018 and 2020. (Lawrence 2018). These observations impacted the mathematical model as an increase in immigrants will impact the projected populations, causing an increase or decrease in total population growth rate.

3. Humans live to approximately 100 years (Vaupel 2010); therefore, a 21 x 21 matrix is reasonable. From this, the age classes for the Leslie matrix were split into 5-year categories. Hence, each new generation produced by the Leslie matrix represents a 5-year gap. This observation is relevant as it impacts the scope mathematical model by investigating only eight new generations.

Observations

1. In 2020, Australia's population was 50.2% female and 49.8% male (ABS, 2020), which may be relevant as Leslie matrices only consider females.

2. In 2017, people aged 85 and over made up 1.76% of Queensland's population (ABS, 2018). Queenslanders between 85-100 years of age show almost identical fertility rates, although their death rates vary ('Centenarians now Australia's fastest-growing demographic', 2021).

3. There was a less than 1% chance of pregnancy for women over 50 years of age (Pesce, 2016) and birth rates for girls under the age of 15 years were "unstable" due to low numbers (Marino, 2016). Birth rates of women aged outside of 15-49 years old were thus low and inconsistent.

4. As shown in Appendix C, the death rates of Australians have mostly stabilised over time, due to improved healthcare and technology, with only a minor decrease in the number of yearly female deaths year from 1997-2017 (ABS, 2021).

5. The birth rates of recent immigrants vary due to their different cultural and geographical backgrounds, while the mortality rates of immigrants should be similar to that of long-term residents as they have the same healthcare access (Peri, 2020).

6. As shown in Appendix D, the immigration rate fluctuates from 2003-2017 before stabilising into a linear trend with a gradient of 160.22 (calculated using excel), meaning the net immigration of Queensland changes over time.

2.2 Assumptions

- An assumption is something that the student is assuming. This assumption is NOT mentioned in the question and is a CHOICE the student is making. The existence of this assumption should have a material effect on the validity of the mathematics.
- It is important that you explain why the assumption is important. Key words to use in this section are "Because" or "therefor".
- If you have looked something up, **use in text referencing to show your source.** Showing sources is one of the best ways to show "documentation".

The QCAA have this to say from the 2022 Mathematical Methods Subject report

> – responses that demonstrate 'documentation of appropriate assumptions' not only include assumptions related to the student's model/solution but also evidence to support the assumption, e.g. the likely effect of an important assumption and how this is considered in the model/solution, or the impact of not making the assumption

QCAA Exemplars from 2022 Specialist Mathematics

Assumptions

1. It was assumed that birth rates would only impact women aged 15-44 as in Australia. The average woman's reproductive years are between ages 15 and 44 (Watson, 2018). This assumption restricts birth rates to only six of twenty-one age classes. The assumption was made to reduce the anomalies to develop clean data.
2. It was assumed that the male-to-female ratio, survival, and birth rate were the same and remained constant for the Western Australian and immigrant population. This was required to restrict data from different years from impacting the trend and the investigation parameters in order to obtain relevant projections. As only a small number of immigrants are introduced into Western Australia each year, they will have minimal impact on the survival, and birth rate (Australian Bureau of Statistics, 2021).
3. It was assumed that the new immigrant population introduced into Western Australia was equally divided into the age classes from 18 to 34. Most migrants to Australia are young adults, with 61.2% aged between 18 and 34 years (Abs.gov.au, 2018). This assumption was made to create a realistic data spread that included the possible impact immigrants' survival and or birth rate would have on the total population.
4. It was assumed that initially, no one left or entered Western Australia prior to immigration. The developed Leslie matrix does not initially account for interstate and international traveling; therefore, this assumption must be made to prevent skewing the data, creating an unrealistic model.
5. It was assumed that the investigation started during 2016 as the female population data collected was from 2016 (Abs.gov.au, 2016). This assumption impacts the investigation as the potential increasing or decreasing growth rate and total population can be compared to secondary data to determine the model's validity.

Assumptions

1. The initial population and birth rates will be multiplied by 0.502 before being used in the calculations (Observation 1). Before graphing, projected populations must then be divided by 0.502 to show the entire projected population, rather than that of females only. Finding the overall gender distribution instead of that of each age group could impact the results' accuracy; however, since the gender distributions are almost identical for each group, this impact should be insignificant.
2. As people aged 85 and over show similar characteristics (Observation 2), it was decided that the final age bracket would be 85+. This may negatively impact the results due to variation in death rates within the age bracket. However, since this age group is small and is the final age group, the assumption will not impact the other age brackets and should have a minimal effect on the model's reasonableness.
3. The fertility rates were only considered for women between the ages of 15-49 years old (Appendix B) (Queensland Government Statistician's Office, 2020). This may negatively impact the results; however, there is too much inconsistency (Observation 3) and not enough information about birth rates outside of this age group to incorporate them effectively.
4. It was assumed that the survival rate and fertility rates were constant. This may impact the accuracy of the results since, in a real-life situation, the death and fertility rates change yearly. Since Queensland's death rates are becoming more consistent (Observation 4), the impact of a constant death rate should be minimal. However, the birth rates over time were inconsistent, with high variation, and thus assuming a constant birth rate may impact the projections' reasonableness.
5. It was assumed that overall immigration would increase by 160.22 immigrants every year (Observation 6). This assumption should improve the results' precision, as it means the model will better represent the growth rate of immigration and hence the overall population projections.
6. While was assumed that overall immigration would increase by 160.22 immigrants every year (Observation 6). This assumption should improve the results' precision, as it means the model will better represent the growth rate of immigration and hence the overall population projections.
7. Covid-19 severely impacted immigration worldwide; however, with Queensland's borders opening soon, immigration rates should soon be normalised (Charumilind et al., 2021). Hence, this assignment was based on pre-pandemic data and disregarded the effects of Covid-19. While this assumption will negatively impact the short-term projections' accuracy, it should improve the long-term projections' reasonableness.
8. As well as base (0) immigration, the three levels of initial net immigration investigated were low (29 050 per year), medium (37 350 per year) and high (46 650 per year). These were multiplied by 0.59, the percentage of female Queensland immigrants in 2017 (ABS, 2019), giving low (17 139.5), medium (22 036.5) and high (27 523.5) initial immigration values.
9. In a Leslie matrix, all individuals within an age bracket are assumed to have the same characteristics. The effect of this assumption on the model's reasonableness was minimised by using small five-year age brackets; however, this could be improved by further reducing the size of each age bracket.

2.3 Mathematical Concepts and Techniques

- This section is not strictly necessary, but many students and teachers find it useful as a way to demonstrate this dot point of the ISMG.

The student work has the following characteristics:	Marks
• documentation of appropriate assumptions • accurate documentation of relevant observations • accurate translation of all aspects of the problem by identifying mathematical concepts and techniques.	3–4

- In this section, students identify the mathematical concepts and techniques that they have learned and explain how this maths will be used to solve this specific problem.

The QCAA explains in the 2022 Specialist subject report that:

> • in the Formulate criterion, judgments about the translation of the problem were supported by identifying the breadth and depth of the student's comprehension of how concepts are applied to develop the mathematical model or representation

- Think… 'What maths will I be using throughout?' and succinctly outline these to the reader (greater detail can be provided as necessary throughout the 'Developing a Solution' section).
- This is also an opportunity for students to demonstrate their technical and procedural vocabulary (more on this later). Use the "maths words" that you've learned in Unit 3. This will help you achieve this dot point from the ISMG.

The student work has the following characteristics:	Marks
• correct use of appropriate technical vocabulary, procedural vocabulary and conventions to develop the response • coherent and concise organisation of the response, appropriate to the genre, including a suitable introduction, body and conclusion, which can be read independently of the task sheet.	3–4

The QCAA defines these words as:

technical vocabulary	terms that have a precise mathematical meaning (e.g. categorical data, chain rule, decimal fraction, imaginary number, log laws, linear regression, sine rule, whole number); may include everyday words used in a mathematical context (e.g. capacity, differentiate, evaluate, integrate, order, property, sample, union)

procedural vocabulary	instructional terms used in a mathematical context (e.g. calculate, convert, determine, identify, justify, show, sketch, solve, state).

You can think of technical vocabulary as "mathematical nouns", and procedural vocabulary as "mathematical verbs".

3. Developing a Solution.

This section is your opportunity to fulfil the following criteria from the ISMG

The student work has the following characteristics:	Marks
• accurate use of complex procedures to reach a valid solution • discerning application of mathematical concepts and techniques relevant to the task • accurate and appropriate use of technology.	6–7

And also…

The student work has the following characteristics:	Marks
• evaluation of the reasonableness of solutions by considering the results, assumptions and observations • documentation of relevant strengths and limitations of the solution and/or model • justification of decisions made using mathematical reasoning.	4–5

You can set this section out however you like, but I like to use sub-sections, showing each step in my solution. Example:

3.1 Plotting Points on Cartesian Plane

3.2 Cannonball Landing Position

3.3 Adjustment to Model

3.4 etc

In communicating your solution, you should use **ALL** of the following:

- Words
- Equations
- Diagrams
- Tables
- Screenshots of Technology use

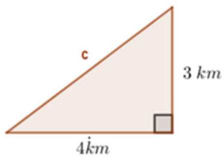
Advice from me: There is a flow to a good Developing a Solution section.

Justify in WORDS what you're about to do.	→	Do some maths using: • Equations • Graphs • Tables • Technology	→	Make observation about what you just found OR Justify whether it's reasonable or not	→	Go back to the start of this cycle and do the next step of your solution

Here is a simple example of what this looks like in practise.

First the length of the hypotenuse must be calculated. This length will be the amount of steel cable required for the flying fox construction.

$$c^2 = a^2 + b^2$$
$$= 4^2 + 3^2$$
$$= 16 + 9$$
$$= 25$$
$$c = \sqrt{25}$$
$$= 5\ km$$



← Explain what you're doing and justify WHY you're doing it.

← Do some maths. This example is an equation and a diagram. But others might be tables, graphs or screenshots of technology use

The length of the cable is 5 km. This result is reasonable, as the hypotenuse found is longer than each of the shorter sides and is shorter than the sum of the shorter sides (7 km) which would not be possible.

← A sentence about what we've found. In this example, I evaluate the reasonableness of the solution (that's an ISMG dot point). I could have also made an observation from my maths (also an ISMG dot point).

Using this length, the cost of the cable can now be calculated...

← The process starts again with this sentence for the next step of my solution.

Use of technology: Make sure you show evidence of your use of technology. Screenshots are your friend.

QCAA samples

Excerpt 5

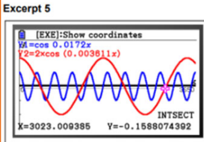


Figure 9:

$x: \cos(0.0172t) = 2 \times \cos(0.003611t)$

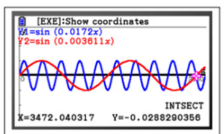


Figure 10:

$y: \sin(0.0172t) = \sin(0.003611t)$

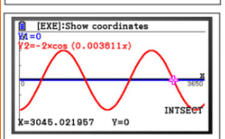


Figure 11:

$z: 0 = -2 \times \cos(0.003611t)$

These observed points of Intersection were then entered into Excel, where the filter function was used to highlight those values with the smallest differences, shown below.

x	y	z
472.5358	462.3729	435.0031
2122.817	1962.458	2175.016
3023.009	3170.124	3045.022

Excerpt 6

To verify this a larger Excel spreadsheet was created to solve the parametric equations for a variety of t values (also extending to ten years):

t	Xleft	Xright	Difference	Intersect?		Yleft	Yright	Difference	Intersect?		Zleft	Zright	Difference	Intersect?
0.5	0.999963	1.999997	1.0000337	FALSE		0.0086	0.001805	0.0067944	FALSE		0	-2	1.999997	FALSE
1	0.999852	1.999987	1.0001349	FALSE		0.017199	0.003611	0.0135882	FALSE		0	-1.99999	1.999987	FALSE

=COS(0.0172*A2) =2*COS(0.003611*A2) =ABS(B2-C2)

Figure 12: Spreadsheet

The Excel equation inserted in the 'Intersect?' column (Figure 13), calculated if the difference between the two sides of the parametric equation was less than 4.2587×10^{-5}, the conversion of Earth's radius to AU, to ensure constant units.

=IF(D2<(0.0000425875),"YES")

The 2022 Specialist Subject report says this about demonstrating your use of technology.

- in the Solve criterion, 'use of technology' must go beyond simple computation or word processing; it must be a key component in the development of the solution to the problem. Evidence of accuracy and appropriateness should be identified and included in the body of the response, e.g. screenshot of excerpt from spreadsheet showing formulas used

4. Evaluate, Verify and Improve

4.1 Evaluating and Verifying

This section is our attempt to hit the following criteria from the ISMG.

The student work has the following characteristics:	Marks
• evaluation of the reasonableness of solutions by considering the results, assumptions and observations • documentation of relevant strengths and limitations of the solution and/or model • justification of decisions made using mathematical reasoning.	4–5

It should be noted if you did a good job in the solve section, you've already been doing some evaluation of your solutions. This section is another chance to get that done.

The QCAA says (methods subject report 2022)

> within the Evaluate and verify criterion, evaluation of the reasonableness of solutions requires consideration of 'results, assumptions and observations' (not just statements about possible reasonableness or unreasonableness of the solution), e.g.
>
> – responses need to demonstrate that the results, assumptions and observations have been considered to appraise and justify the solutions. Any assumptions and observations introduced throughout the report could be used while evaluating the reasonableness of solutions
>
> – the evaluation could include the use of technology to verify solutions or the use of both mathematical and everyday language to justify solutions.

My advice:

1. You **MUST** evaluate your solution in **THREE** ways:
   a. By considering your results. This can be done by:
      i. Using Technology or an alternative method to VERIFY your solution
      ii. Calculating error against real-world data
   b. By considering your assumptions. (see exemplar below)
   c. By considering your observations. (see exemplar below)

This exemplar from the 2022 Specialist Subject report (with my comments added down the left) show a great way to achieve this.

Graph 3 displays a comparison between the two models:

Both total population predictions follow a linear trend, both with R^2 values greater than 0.99. The linear nature of the models reinforces their validity as it is evident there are no drastic or unrealistic changes in population over the 100-year period, as would be expected. However, it is evident this model would eventually fail as the total population would soon dip below zero:

$$y = -0.8792x + 1899.8$$

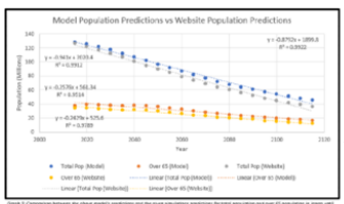
$$Substitute\ y = 0$$

$$0 = -0.8792x + 1899.8$$

$$-1899.8 = -0.8792x$$

$$x = \frac{-1899.8}{-0.8792}$$

$$x = 2160.8$$



Do some maths. Many students make the mistake of jumping straight to just paragraphs of explanation. Evaluation should involve some numbers and comparison of those numbers. This maths sometimes appears at the start.

It can be seen in *Graph 4* that the patterns followed by the two sets of data are very similar, both increasing until the year 2055 before dropping off slightly. While the model then plateaus at around 36.3% after the year 2075, the website simulation predicts a small spike around the year 2090 before plateauing again at approximately 33.3%. The final difference in percentages is just 3.05%, or a 9.15% error, again very similar results given the large time-period. There is, however, several reasons for the variation between models:

This paragraph interprets the maths done above to evaluate the reasonableness of the solution by considering the results

Leslie matrices are one of the best methods for predicting population change amongst a certain population, however they require a variety of assumptions to become viable (Jones, 2008). Firstly, it was observed the model did not account for immigration and emigration; it was therefore assumed this did not impact the population in any way. However, this is an unreasonable assumption to make, as emigration and immigration are inevitable in countries such as Japan and would influence their populations, hence decreasing the reasonableness of the model.

The assumption that the birth and survival rates did not change over the 100-year period was again unrealistic because factors such as medicine and fertility rates amongst populations are constantly changing. Additionally, it was assumed the rounded values from the online simulation would have no impact on calculations and results, however, after having the numbers manipulated and the small uncertainties in rounding compounded over a 100-year period, disparities occurred, reducing the reasonableness of the model.

These two paragraphs evaluate the reasonableness of the solution by considering the assumptions that were made

When calculating the total populations from the female populations the proportion used (51.17% of population is female) was found through research. However, the original data presented by the online simulation suggested the proportion was 51.51% female and hence the female values should have been multiplied by $\frac{1}{0.5151} = 1.9411$ rather than the original 1.95427. This would have had minimal impact on the conclusions drawn, however, could have some impact on the small discrepancies between the two models.

A paper published by the JIPSSR estimates the Japanese population will be approximately 40 million in the year 2115, an over 65 population of roughly 14 million, and hence a percentage of the population over the age of 65 at about 35% (IPSS, 2002). The devised model suggests the population in the year 2115 will be approximately 45.5 million, an over 65 population of roughly 16.5 million, and hence a percentage of the population over the age of 65 at about 36%. This 1% discrepancy between the estimates of the JIPSSR and the Leslie matrix model after 100 years indicates a high degree of reasonableness and reliability.

These two paragraphs evaluate the reasonableness of the solution by considering the observations that were made

4.2 Strengths and Limitations

This is the dot point we're trying to hit in this section

The student work has the following characteristics:	Marks
• evaluation of the reasonableness of solutions by considering the results, assumptions and observations • documentation of relevant strengths and limitations of the solution and/or model • justification of decisions made using mathematical reasoning.	4–5

Students get confused about what Strengths and limitations are. The QCAA says (2020 Specialist Subject report).

- A strength is an aspect or feature of the solution and/or model that makes it useful. A limitation is an aspect or feature of the solution and/or model that limits its usefulness; a weakness. Limitations are often a direct consequence of assumptions (making less restrictive assumptions gives opportunities for refinement).

My Advice: I see these often enough that I must tell you they **ARE NOT** strengths or limitations. **DO NOT WRITE ANYTHING LIKE THIS.**

- ~~Technology made the maths easier.~~
- ~~I couldn't do the maths.~~

The Exemplar from QCAA 2022 Specialist subject report gives good examples.

Strengths	Limitations
The model was tested up to 100 years into the future and remained similar to 'The Habitable Planet' prediction (within 9% error in percentage of population over 65) over the 100-year time-period. Additionally, the model was corroborated by the JIPSSR, with just a 1% difference in prediction of percentage of population over the age of 65.	The model relies on the rounded data provided by 'The Habitable Planet'. These rounded values will have had implications on the conclusions drawn after 20 time-steps of matrix multiplication.
The model uses a 5-year time step to accurately identify the trend that best fits the data and make the most reliable predictions on future populations.	As discussed above, many unreasonable assumptions, such the birth and survival rates will not change over time, immigration and emigration have no impact, and exactly 50% of births are female, are unrealistic and limit the reasonableness of the model.
The linear trends identified verified there were no drastic changes to the course of the total population as would be expected in a large population.	The linear trend identified is only accurate for the given time-period, because as seen above, the population will theoretically reach negatives. If the matrix were to be continued further into the future, a more accurate trend would have been identified.
The required data for the Leslie matrix was successfully extracted from the simulation, such as female birth rate, and the survival rates of the individual age demographics.	

5. Conclusion.

Do not focus a large amount of energy on this. It must exist though, because we need to hit this dot point.

The student work has the following characteristics:	Marks
• correct use of appropriate technical vocabulary, procedural vocabulary and conventions to develop the response • coherent and concise organisation of the response, appropriate to the genre, including a suitable introduction, body and conclusion, which can be read independently of the task sheet.	3–4

6. Reference List

This should exist, because you did research and in-text referencing to have decent observations and assumptions.

7. Appendixes

THIS ABSOLUTELY WILL NOT BE MARKED. Nothing you put here contributes to your overall grade.

The QCAA says (2022 Specialist Subject report).

> - As appendixes are not to be marked, students should provide evidence relevant to performance descriptors (e.g. use of complex procedures, use of technology) in the body of the report. Large datasets, evidence of authentication (e.g. creating a piece of code, completing an experiment), or repeated calculations can be put in an appendix.

Anything too large to be in the body of the report should be placed in the appendix. Each item in the appendix should be numbered Appendix 1, Appendix 2 etc and referred to as such in the body of the report.

That's the end. Good Luck.

Joel Speranza